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Games, graphs, and machines

Equivalence relations

July 30, 2024

Reminders

1. Office hours: Tue 10:30-11:30 and Fri 11:30-12:30 (HN 4.56).
2. Need two class representatives.

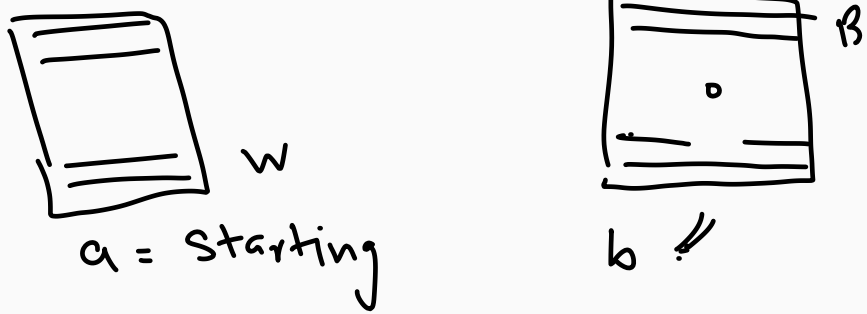
Equivalence relation or not?

Is $\sim$ an equivalence relation on S? Reflexive, symmetric, transitive

1. $S = \mathbb{R}$ and $a \sim b$ if $|a - b| < 1$.
2. $S =$ States of a chess-board and $a \sim b$ if we can reach b from a by a sequence of legal moves.
3. States of a rubik's cube. $\Rightarrow$ eqv. relation

1. Fails transitivity
   e.g. $2 \sim 2.9$, $2.9 \sim 3.8$
   $2 \not\sim 3.8$.

2.



Equivalence relation or not?

Is $\sim$ an equivalence relation on S?

1. $S = \text{Pow}(X)$ and $A \sim B$ if $|A| = |B|$.
2. $S = \text{Pow}(X)$ and $A \sim B$ if $A \subseteq B$.

$S = \text{Pow}(X)$ X finite

$A \subseteq B$ subset not equal

$A \subseteq B$ ↖ same ↗ $A \subseteq B$

1. ✓ 2. fails symmetry

Ex. $X = \{1,2,3\}$

Eqv. classes of $\sim$ 4 of them

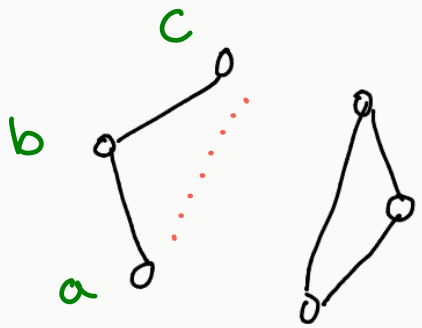
$\{\phi\}$, $\{\{3\},\{2\},\{3\}\}$, $\{\{1,2\},\{1,3\},\{2,3\}\}$

$\{\{1,2,3\}\}$

Graph of an equivalence relation

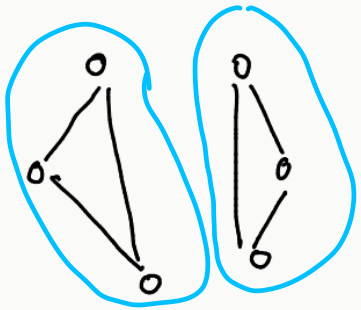
Which of the following is the graph of an equivalence relation?
(Self-loops and arrows omitted)

So assuming reflexive & symmetric



(1)

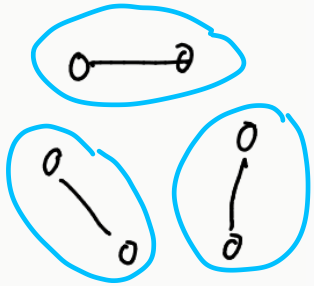
No.



(2)

✓

Eq classes



(3)

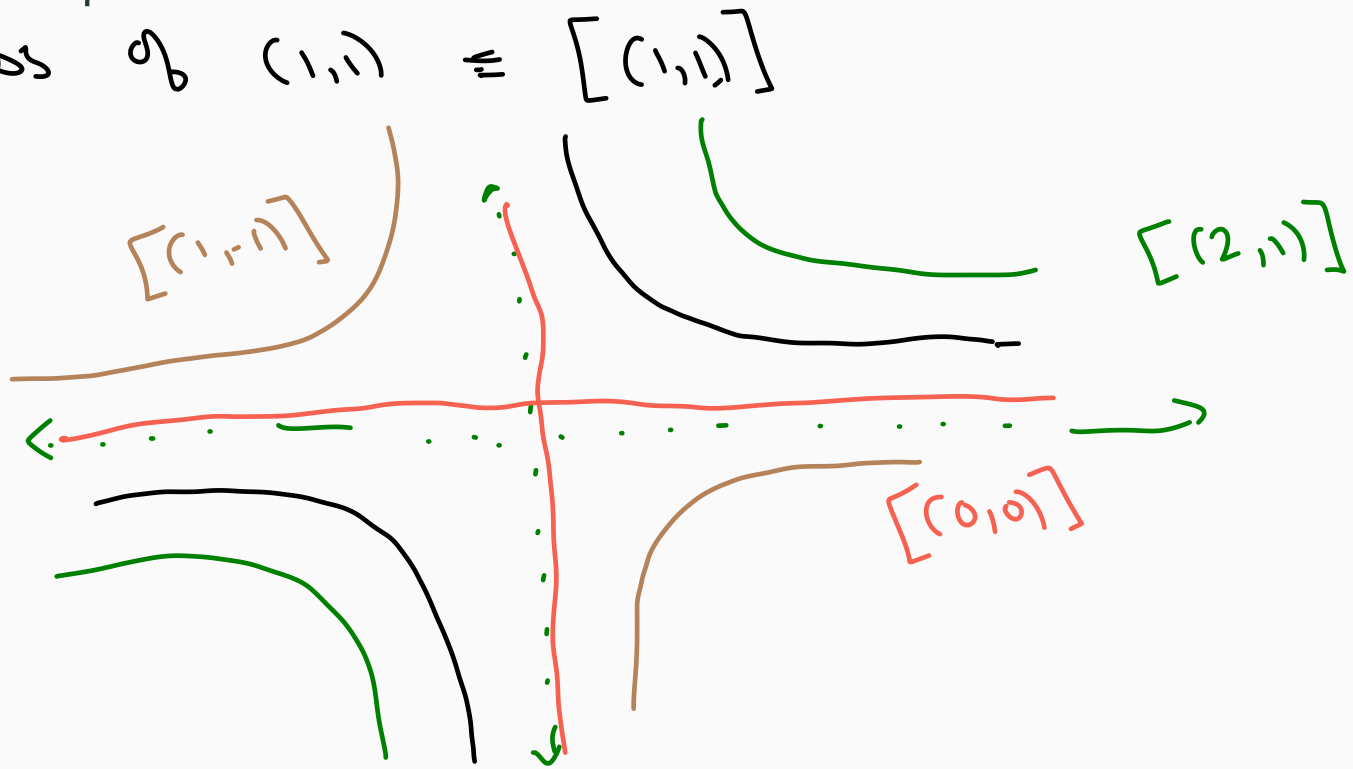
✓

Equivalence classes 1

Let $\sim$ be the relation on $\mathbb{R}^2$ defined by

$$(a, b) \sim (c, d) \text{ if } ab = cd.$$

Describe the equivalence classes.



Equivalence classes 2

Let $\sim$ be the relation on $\mathbb{Z}$ defined by

$$(a, b) \sim (c, d) \text{ if } ab \text{ is a square.}$$

Describe the equivalence classes.

Skipped

Equivalence classes and partitions

Let $\sim$ be an equivalence relations on S. Convince yourself that the equivalence classes *partition* S:

- any two distinct equivalence classes are disjoint
- the union of all equivalence classes is S.

Every elt of S is an elt of exactly one eqv class.

Equivalence relation defined by a function

Let $f : S \to T$ be a function. Say $a \sim b$ if $f(a) = f(b)$.

1. Is this an equivalence relation?
2. Describe the equivalence classes for $f : \mathbb{R}^2 \to \mathbb{R}$ defined by $f(a, b) = ab$.